An extension of an inequality of Miranda for Hölder continuous functions.

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In this seminar, we focus on a sufficient condition for a real valued continuously differentiable function f in an open subset Ω of $\mathbb{R}^n$ to be α - Hölder continuous in Ω for some $\alpha \in]0,1]$.

If $\Omega =]0,1[$, then an elementary sufficient condition is that $|t|^{1-\alpha}|f'(t)|$ is bounded for $t \in]0,1[$.

If $\Omega = \mathbb{B}_{n-1}(0,1) \times]0,1[$, then an elementary sufficient condition is that $|t|^{1-\alpha}|\nabla f(x,t)|$ is bounded for $(x,t) \in \mathbb{B}_{n-1}(0,1) \times]0,1[$ (cf. [1]).

If $\gamma \in C^1(\overline{\mathbb{B}_{n-1}(0,r)},]-\delta,\delta[)$ for some $r, \delta \in]0,\infty[$ and if $\Omega = \text{hypograph}_s(\gamma)$, then Miranda (cf [6]) has shown that a sufficient condition for the α - Hölder continuity of $f \in C^1(\Omega)$ is that $|\gamma(x) - t|^{1-\alpha}|\nabla f(x,t)|$ is bounded for $(x,t) \in \text{hypograph}_s(\gamma)$.

In this talk, we present the extension of Miranda's inequality based on book draft[4], for the treatment in the case where Ω is a Lipschitz set, we refer to [5].

References

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