An interpolation result for Sobolev spaces

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We present a Theorem, first proved by R.DeVore and K.Scherer in [3], which tells us that through the real K-method, see [1], it is possible to obtain classical Sobolev spaces $W^{k,p}(\mathbb{R}^n)$ by interpolating the spaces $W^{k,1}(\mathbb{R}^n)$ and $W^{k,\infty}(\mathbb{R}^n)$. This result is a generalization of the well-known formulation of Lorentz spaces $L^{p,q}(\mathbb{R}^n)$ as interpolation of $L^1(\mathbb{R}^n)$ with $L^\infty(\mathbb{R}^n)$ by means of the real K-method. In a way that recalls this very last result, the proof of the aforementioned Theorem is obtained by proving a proportionality between the K functional for a function $f \in W^{k,1}(\mathbb{R}^n) + W^{k,\infty}(\mathbb{R}^n)$ at a prescribed $t > 0$ and the sum of the $L^1(0,t)$-norms of the decreasing rearrangements of the weak derivatives of order $\leq k$ of f. Such a proportionality will be first proved for splines of order k over a decomposition in cubes of $\mathbb{R}^n$, and with a density argument, see [2], will be then generalized to a a generic function $f \in W^{k,1}(\mathbb{R}^n) + W^{k,\infty}(\mathbb{R}^n)$.

References

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