

Selmer Group and Tate-Shafarevich Group

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Abstract

In the study of the arithmetic of elliptic curves, the Selmer groups and the Tate-Shafarevich groups play an essential role. In fact, we would like to compute the Mordell-Weil group efficiently which can be reduced to the problem of computing weak Mordell-Weil group. One can embed the weak Mordell-Weil group into an effectively computable group and investigate the "error term". More precisely, let $\phi : E/K \rightarrow E'/K$ be an isogeny of elliptic curves defined over a field K , the Selmer groups $S^{(\phi)}(E/K)$ and the Tate-Shafarevich group $\text{III}(E/K)[\phi]$ fit into the following exact sequence

$$0 \longrightarrow E'(K)/\phi(E(K)) \longrightarrow S^{(\phi)}(E/K) \longrightarrow \text{III}(E/K)[\phi] \longrightarrow 0$$

Hence, Tate-Shafarevich groups measure the extent to which the Hasse principle fails to hold for the elliptic curve. In this seminar, I will explain how one tries to calculate there groups follows chapter X from Silverman's book.

References

- [1] Joseph H. Silverman, *The Arithmetic of Elliptic Curves*, Springer-Verlag New York, 2009
- [2] Takeshi Goto, *A Study on The Selmer Groups of Elliptic Curves With a Rational 2-Torsion*, Doctoral Thesis
<https://www.ma.noda.tus.ac.jp/u/tg/files/thesis.pdf>