

t-structures in Triangulated Categories: The Abelian Heart and Relations with Torsion Pairs

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After a brief introduction on triangulated categories, we will give the definition of t-structure and show the standard example in the derived category of an Abelian category.

A t-structure $(\mathcal{U}, \mathcal{V})$ is naturally endowed with two truncation functors $\tau_{\mathcal{U}}$ and $\tau_{\mathcal{V}}$. We will prove this fact and state some basic properties of these functors.

The heart of a t-structure is the subcategory $\mathcal{A} = \mathcal{U} \cap \mathcal{V}[1]$. It turns out that this is an Abelian subcategory of the triangulated category, and we will show a sketch of the proof.

Given a t-structure $(\mathcal{U}, \mathcal{V})$, one can notice that also the shift $(\mathcal{U}[1], \mathcal{V}[1])$ is a t-structure. The composition of the truncation functors $\tau_{\mathcal{U}}$ and $\tau_{\mathcal{V}[1]}$ defines a cohomological functor $H : \mathcal{D} \rightarrow \mathcal{A}$ from the triangulated category \mathcal{D} to the heart of the t-structure \mathcal{A} .

In the last part of the seminar we will introduce torsion pairs in Abelian categories. We will follow the construction by Happel-Reiten-Smalø to show that given a torsion pair $(\mathcal{T}, \mathcal{F})$ in an Abelian category \mathcal{A} , this defines a t-structure $(\mathcal{U}_{\mathcal{T}}, \mathcal{V}_{\mathcal{F}})$ in the derived category $\mathcal{D}(\mathcal{A})$. The heart of this latter t-structure is the Abelian category $\mathcal{A}_{(\mathcal{T}, \mathcal{F})}$ and the pair $(\mathcal{F}[1], \mathcal{T})$ is a torsion pair in this category.

REFERENCES

Dragan Miličić. Lectures on Derived Categories, <http://www.math.utah.edu/milicic/Eprints/dercat.pdf>

D. Happel, I. Reiten, S. Smalø. Tilting in Abelian Categories and Quasitilted Algebras, *Memoirs of the American Mathematical Society*, 575 (1996).