

Multiplicative lattices and pre-Lie algebras

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Abstract

Lattices appear naturally in different algebraic structures: if we have a group G we can consider its lattice of normal subgroups, for a ring R we have its lattice of ideals, and so on. But these structures often present other natural operations worth considering, hence the need of defining multiplicative lattices.

A multiplicative lattice is, as the name suggests, a lattice on which we define a further operation, called multiplication. By doing so, we get a tool to describe different well-known situations, in particular we will define the Zariski spectrum and see how it changes depending on the multiplication we choose.

As an application, we will define a class of non (necessarily) associative algebras, called pre-Lie algebras, we will see their basic properties then proceed to define an adequate multiplication on its lattice of ideals. In order to do so, Huq and Smith commutators will be introduced, and in particular we will show that they coincide on such algebras.

References

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